Performance Model for Unbound Granular Materials in Pavements

Licentiate Thesis

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Abstract

Recently, there has been growing interest on the behaviour of unbound granular material in road base layers. Researchers have studied that the design of a new pavement and prediction of service life need proper characterization of unbound granular materials, which is one of the requirements for a new mechanistic design method in flexible pavement.

Adequate knowledge of the strength and deformation characteristics of unbound layer in pavements is a prerequisite for proper thickness design, residual life determination, and overall economic optimization of the pavement structure. The current knowledge concerning the granular materials employed in pavement structures is limited. In addition, to date, no general framework has been established to explain satisfactorily the behaviour of unbound granular materials under the complex repeated loading which they experience.

In this study, a conceptual method, packing theory-based model is introduced; this framework evaluates the stability and performance of granular materials based on their packing arrangement. In the framework two basic aggregate structures named as Primary Structure (PS), and Secondary Structure (SS). The Primary Structure (PS) is a range of interactive grain sizes that forms the network of unbound granular materials. The Secondary Structure (SS) includes granular materials smaller than the primary structure. The Secondary Structures fill the gaps between the particles in the Primary Structure and larger particles essentially float in the skeleton.

In this particular packing theory-based model; the Primary Structure porosity, the average contact points (coordination number) of Primary Structure, and a new parameter named Disruption Potential are the key parameters that determine whether or not a particular gradation results in a suitable aggregate structure.

Parameters mentioned above play major role in the aggregate skeleton to perform well in terms of resistance to permanent deformation as well as load carrying capacity (resilient modulus). The skeleton of the materials must be composed of both coarse enough and a limited amount of fine granular materials to effectively resist deformation and carry traffic loads.

**Keywords:** Unbound granular materials; Aggregate; Packing theory; Gradation; Primary Structure; Secondary Structure; Permanent deformation; Resilient modulus.
Dedication

To my new upcoming baby
Acknowledgements

I am heartily thankful to my supervisors, Prof. Björn Birgisson, Asst.Prof. Denis Jelagin and Dr. Alvaro Guarin whose encouragement, guidance and support from the initial to the final level enabled me to develop an understanding of the subject.

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I owe a great many thanks to Mrs Agneta Arnius who helped and supported me in all administrative issues since I joined this department.

For more reasons than one, I could not have completed this project without the support of my loving family. I thank my wife, Zini for her daily motivation to complete this thesis. The constant encouragement of my mother and brothers in Ethiopia has also provided me to be more committed in my educational life.

Lastly, I offer my regards and blessings to all my colleagues who supported me in any respect during the completion of the Thesis.

Tatek F. Yideti

Stockholm, June 2012
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<th>Definition</th>
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<td>PS</td>
<td>Primary Structure</td>
</tr>
<tr>
<td>SS</td>
<td>Secondary Structure</td>
</tr>
<tr>
<td>DASR</td>
<td>Dominant Aggregate Size Range</td>
</tr>
<tr>
<td>AASHTO</td>
<td>American Association of State Highway and Transportation Office</td>
</tr>
<tr>
<td>USCS</td>
<td>Unified Soil Classification System</td>
</tr>
<tr>
<td>DP</td>
<td>Disruption Potential</td>
</tr>
<tr>
<td>DF</td>
<td>Disruption Factor</td>
</tr>
<tr>
<td>DF_T</td>
<td>Total Disruption Factor</td>
</tr>
<tr>
<td>V_{PS}</td>
<td>Volume of PS Primary Structure aggregates</td>
</tr>
<tr>
<td>V_{SS}</td>
<td>Volume of PS Secondary Structure aggregates</td>
</tr>
<tr>
<td>V_{agg&gt;PS}</td>
<td>Volume of aggregates bigger than Primary Structure</td>
</tr>
<tr>
<td>DM</td>
<td>Disruptive Materials</td>
</tr>
<tr>
<td>V_{DM}^{SS}</td>
<td>The volume of disruptive material</td>
</tr>
<tr>
<td>V_{free}^{PS}</td>
<td>The free volume within the primary structure</td>
</tr>
<tr>
<td>V_{PDM,o}^{PDM}</td>
<td>The volume of disruptive materials for octahedral structure</td>
</tr>
<tr>
<td>V_{PDM,t}^{PDM}</td>
<td>The volume of disruptive materials for tetrahedral structure</td>
</tr>
<tr>
<td>V_{v,PS,o}</td>
<td>The volumes of voids in PS for octahedral structure</td>
</tr>
<tr>
<td>V_{v,PS,t}</td>
<td>The volumes of voids in PS for tetrahedral structure</td>
</tr>
<tr>
<td>CCP</td>
<td>Cubic Close Packing</td>
</tr>
<tr>
<td>HCP</td>
<td>Hexagonal Close Packing</td>
</tr>
<tr>
<td>2-D</td>
<td>Two Dimensional</td>
</tr>
<tr>
<td>3-D</td>
<td>Three Dimensional</td>
</tr>
<tr>
<td>SST</td>
<td>Sand Stone</td>
</tr>
<tr>
<td>CONC</td>
<td>Crushed Concrete</td>
</tr>
</tbody>
</table>
Gr and GR  Granite
S&G  Sand and Gravel
$D_{w,\text{avg}}$  Weighted average of two consecutive sieve sizes
$d_{w,\text{avg}}$  Weighted average of void size of two consecutive sieve sizes
$\phi_1$ and $\phi_2$  The percentage retained in two consecutive sieve sizes
r  The radius of sphere
a  The length of the cubical structure
$D_1$ and $D_2$  The two consecutive sieve sizes
H  The diagonal length in cubical structure
h  The distance between particles
ASTM  American Society for Testing and Materials
US STD  United State Standard
BS  British Standard
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List of Publications

Paper I
Tatek F. Yideti, B. Birgisson, D. Jelagin and A. Guarin, “Packing theory-Based Framework to evaluate Permanent deformation of Unbound Granular Materials”


Paper II

1.0 INTRODUCTION

1.1 Background

Unbound granular base and subbase layers have a major impact on the long term performance and load carrying capacity of pavements. Aggregate size distribution plays a key role on the performance of granular materials; this relationship must be understood in order to improve pavement design and construction procedures (Thom and Brown, 1988; Dowson et al., 1996 and Kolisoja, 1998, Lekarp, 1999, and Ekblad, 2007).

In this study, particle size distribution, packing arrangement, disruption of coarse aggregate load carrying fraction, porosity, and contact points per particle are used to evaluate permanent deformation, resilient modulus and strength of unbound granular materials.

1.2 Objectives

The main objectives of this study are:

- Develop a generalized packing theory-based framework for unbound materials
- Identify the load carrying aggregate particles from the aggregate size distribution
- Investigate the stability nature (Disruption Potential) of load carrying aggregate particles
- Study the porosity and contact points of load carrying aggregate structure
- Evaluate mechanical properties of unbound materials (resistance to permanent deformation and resilient modulus) based on the developed framework.

1.3 Scope

The proposed framework assumes the shape of the particles to be spherical and the density of the materials to be constant throughout each sieve size. The validation of the framework has been done on nineteen unbound base and subbase materials with different mineralogical composition. The performance data were taken from previously conducted repeated loading triaxial test (RLTT). The framework considered only packing and grain size distribution, not seen further the shape, surface texture and angularity of materials. This model is generally applicable to any type of granular materials regardless of their sieve size standards.
2.0 LITERATURE REVIEW

2.1 Unbound Granular Material

Unbound Granular Material (UGM) is a frictional soil with different particle shapes and sizes; UGM is an inhomogeneous and non-isotropic material. It transmits traffic loading to the subgrade by reducing the vertical compressive stress. The performance of pavements is to a great extent defined by the characteristics of the unbound base and subbase layers. Typically in Sweden, the conventional natural gravel and crushed stones can be found in a quarry site with abundance of granite and limestone (Figure 1).

**Figure 1 Unbound Granular Material**

Unbound granular materials can be characterized based on their physical properties such as gradation, plasticity, hardness, durability, and on their shear strength properties. Particle size distribution of the granular material affects the most fundamental properties of unbound granular material such as resilient modulus, permanent deformation, durability, and permeability (Thom, N and Brown, S., 1988). Hicks and Monismith (1971) observed that the angularity of crushed aggregate provided a higher resilient modulus than uncrushed gravels with angular or sub-angular shaped particles. In order to establish more rational pavement design and construction, as well as prediction of service life, the effect of grain size distribution on the performance of unbound granular materials must be better understood.

2.2 Mechanical properties of Unbound Granular Materials

The mechanical properties of unbound granular materials comprise their stiffness, stability and load-bearing capacity. The stability of unbound granular materials is expressed by the permanent deformation behaviour; it can be defined as a measure of the ability to resist permanent deformation.
Resilient modulus exhibits an increasing trend with increasing dry density (Robinson, 1974). Trollope (1962) showed that the influence of density on resilient modulus for particularly uniform sand materials increased up to 50% for loose to dense specimens.

The deformation resistance of unbound granular materials depends on the applied stresses, which is dependent on the physical factors such as gradation and fines content; the effect of grading and compaction on plastic strain was also studied by Thom and Brown (1988); they concluded that the resistance to permanent strain decreased when the specimens were not compacted well for all grading types.

Barksdale and Itani (1989) studied the effect of the plasticity of fines on the deformation of granite gneiss and reported that an increase in the fines content from 0 to 10% increased the resilient modulus by 60%. In addition, they found that as the fines content increased, the amount of permanent strain also increased. Dowson et al. (1996) established that the resistance to plastic strain was higher for the densest material; he also determined that the effect of grading on plastic strain is more considerable than the degree of compaction. Kolisoja (1997) observed 20% reduction in resilient modulus by adding another material as the fines fractions. Kolisoja (1998) also argued that substantially higher permanent strain may be expected for aggregates containing extremely high fines content or at a low content of fines. He also concluded that a decrease in void content increases the number of contact points per particle with other particles (coordination number), which increases the stiffness of the material. Correspondingly, Hoff (1999) reported the maximum grain size also affects both the fundamental properties; resilient and permanent deformation of unbound granular materials. The results showed that bigger maximum grain size exhibited lower resistance to deformation.

Similarly, Lekarp, et al. (2000) presented a “state-of-art” on the permanent strain response of unbound aggregates and concluded that the effect of increase in fines content has an inverse effect on the permanent deformation resistance in granular materials and they observed an increase in density highly improved the resistance to permanent strain. The mineralogical composition and the internal structure of the particles also have a considerable impact on deformation properties (Arm, 2003). She suggested that in order to resist external load actions as well as for deformation properties to remain the same over the life of the road, the materials particle size and shape must not be changed. Additionally, Ekblad (2007) evaluated one crushed granite aggregate with four different gradations, from Skärlanda, Sweden. The study showed that the resilient modulus decreased with increasing water content and decreasing grading coefficient, i.e., the finer the material the lower the resilient modulus.
2.3 Porosity

Porosity describes the amount of open space in granular materials and is defined as the ratio of the volume of void over the total volume. It is largely influenced by factors as particle size, shape, the uniformity of the grain size, the type of granular materials, and particle interlocking.

2.4 Coordination number of particles

The coordination number, or number of particle contact points with other particles, is an important parameter to describe the geometrical arrangement of particles. It is widely used for evaluating mechanical properties of particles that related to the connectivity between particles. The strength of a mass of unbound granular materials depends on the number of particle contacts at which the capacity to carry load can be ensured.

In general, a densest packing gives a higher coordination number and a loosest packing gives a lower coordination number. A few researchers have related the coordination number to porosity (Smith, et al., 1929; Field, 1963; Gray, 1968; Ching et al., 1991).

2.5 Disruption Factor of aggregates

Roque (2006) developed an asphalt mixture model composed by Dominant Aggregate Size Range (DASR), which is the coarse aggregate load carrying structure, and Interstitial Component (IC), which includes aggregate particles smaller than DASR and binder. They established a procedure for DASR identification and proposed DASR porosity criterion (DASR porosity < 50%).

Guarin (2009) studied the effect of Interstitial Component on asphalt mixture performance; he introduced Disruption Factor (DF), which is a parameter to evaluate the potential of IC particles to disrupt the DASR structure and defined as the ratio of the volume of potentially disruptive IC particles over the volume of DASR void.

\[ DF = \frac{\text{Volume of potentially disruptive IC particles}}{\text{Volume of DASR voids}} \]  

where, volume of potentially disruptive IC particles considers particles smaller than DASR and bigger than the voids of DASR; it is calculated by using volumetric relationships. Volume of DASR voids was determined as a function of the type of DASR structure (cubical or hexagonal) and the number of DASR particles. The type of packing arrangement (simple cubic
or close hexagonal) was selected according to the DASR porosity. The determination of voids in DASR as well as the volume of potentially disruptive IC particles was done based on packing arrangement and volumetric relationships of aggregates.

Guarin (2009) concluded that the volume of potentially disruptive IC particles appears to be a key factor of IC gradation that may help to control asphalt mixture rutting and cracking performance and proposed DF to be included in gradation guidelines for mixture performance.
3.0 THEORETICAL FRAMEWORK

3.1 Primary and Secondary Structure

Unbound materials are composed of stones with a size distribution determined and described through a gradation analysis. In this study, it is stated that different aggregate sizes have different roles in the load carrying and in the long term performance of a pavement structure. Accordingly, the model identifies two basic components of the skeleton of unbound granular materials which are Primary Structure (PS) and Secondary Structure (SS). PS is a range of interactive grain sizes that carry the loads and SS is a range of grain sizes smaller than the PS. PS and SS form the skeleton of unbound granular materials.

3.1.1 Identification of Primary Structure

The identification of primary structure range has been done based on the 3-D densest possible packing (cubic close packing, CCP). The PS in the skeleton is thus composed of particles having a certain size range. Equation (2) is used to identify the size range of the PS for unbound granular materials; the detail procedure is shown in Paper I.

\[
0.217 + 0.515 \left( \frac{D_2}{D_1} \right) D_1 \leq d_{\text{w.avg}} \leq 0.515 + 0.217 \left( \frac{D_2}{D_1} \right) D_1
\]

(2)

where, \( D_1 \) and \( D_2 \) are the two larger and smaller consecutive sieve sizes, \( d_{\text{w.avg}} \) is the weighted average void size.

3.1.2 Primary Structure Porosity

In the framework, primary structure porosity (\( n_{\text{ps}} \)) is one of the key parameters used for evaluating resilient modulus of unbound granular materials; it is defined as the ratio of the volume of voids in the PS over the total volume of granular mix (skeleton). The volume of voids in PS is everything in the skeleton that is not considered to be part of the PS, and the total volume of the granular mix (skeleton) is all except the volume of particles bigger than the Primary Structure. Equation (3) is derived from ordinary porosity formula that is known in geotechnical engineering.

\[
n_{\text{ps}} = \frac{V_{\text{PS}}}{V_{\text{PS}}} * 100\% = \frac{V_{\text{SS}} + V_{\text{v}}}{V_{\text{TGM}} - V_{\text{AGG(\geq PS)}}} * 100\%
\]

(3)
where:

\( V_T \): Volume of total granular mix.

\( V_{agg(L_1)} \): Volume of granular materials retained on L_1 sieve.

\( V_{SS} \): Volume of Secondary Structure (granular materials smaller than PS).

\( V_v \): Volume of voids in granular mix.

### 3.1.3 Primary Structure Coordination number

Porosities and their corresponding visualized theoretical contact points of four packing arrangements (cubic, orthorhombic, tetrahedral, Rhombohedral) were used as initial points to formulate the primary structure coordination number. The equation of coordination number as a function of porosity is developed by fitting the theoretical number of contact points to their corresponding porosities. The coordination numbers for four systematic packing arrangements are shown in Table 1. Equation (4) shows the derived mathematical relation for PS coordination number and PS porosity.

**Table 1 Four Packing Arrangements**

<table>
<thead>
<tr>
<th>Packing Arrangements</th>
<th>Coordination number</th>
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</thead>
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<tr>
<td>Simple Cubic Packing</td>
<td>6</td>
</tr>
<tr>
<td>Orthorhombic</td>
<td>8</td>
</tr>
<tr>
<td>Tetrahedral</td>
<td>10</td>
</tr>
<tr>
<td>Rhombohedral</td>
<td>12</td>
</tr>
</tbody>
</table>

\[
 cn_{ps} = 2.827 \left[ \frac{n_{ps}}{100} \right]^{-1.069} \tag{4}
\]

where, \( cn_{ps} \) is the PS coordination number and \( n_{ps} \) is the PS porosity in percent.
3.2 Disruption Potential

A small percentage of fines fraction (SS) may fail to provide an adequate support to the coarse aggregates (PS), while too high a percentage of fines material may result in PS particles loosing contact with each other, thus reducing load carrying capacity of the materials and eventually the PS aggregates likely to disrupt. In Paper I, a parameter so called, Disruption Potential (DP) shows the ability of SS to disrupt the PS; it can be defined as the ratio of the volume of potentially disruptive fine material over the free (available) volume within the primary structure. The mathematical expression for DP is illustrated in the following Equation (5).

\[ DP = \frac{V_{DM}^{SS}}{V_{free}^{PS}} \]  

where, \( V_{DM}^{SS} \) is the volume of Disruptive Material and \( V_{free}^{PS} \) is the free volume within the primary structure

4.0 MATERIALS FOR VALIDATION

The validation of the framework was done using different unbound granular materials from two different countries (Sweden and USA). Among these, seven materials data were collected from two KTH studied references (Ekblad, 2007 and Lekarp, 1999). These Swedish originated materials composed of granite (Granite 1-4, GR), sand and gravel (S&G) and crushed concrete materials (CONC). S&G material typically used as a subbase materials but the rest materials are mainly used as base course. The experimental data for the remaining thirteen USA granular base materials, including three Louisiana State base materials and ten Missouri base materials are used in the model validation.
5.0 FRAMEWORK EVALUATION

5.1 Effect of Disruption Potential (PAPER I)

Disruption Potential (DP) plays a major role on the resistance to permanent deformation behaviour of unbound granular materials. As shown in Figure 2, it was hypothesized that increasing DP values influences the stone to stone contact between the load carrying particles and leading to higher permanent deformation. On the other hand, when there is too low volume of fines fraction leads to low DP values and there will not be enough fines material that can support the load carrying particles and eventually the granular material composition becomes less resistant to permanent deformation.

![Disruption Potential vs Permanent Deformation](image)

**Figure 2** Relationship between DP and Permanent deformation

5.2 Effect of Secondary Structure on Permanent deformation (PAPER I)

Figure 3 shows the relationship between the normalized permanent strain and the volume of SS. It can be seen that low and high percent volume of SS has a great effect on the deformation behaviour of unbound materials. Good performance is observed for materials with optimum amount of SS that can support the PS in the overall skeleton of granular materials.
5.3 Influence of PS Porosity on the Resilient Modulus (PAPER II)

The PS porosity of the nineteen evaluated materials satisfactorily correlated with their corresponding resilient modulus (Figure 4). It can be observed that increasing the porosity of the primary structure results in a decrease in the resilient modulus. This general trend is common as a matter of fact for any granular material and an increase of pore space within the aggregates tends to lower resilient modulus. There is a stone-to-stone contact between particles when the PS is between 26% and 48%; these porosities correspond to porosities of rhombohedral and simple cubic packing arrangement respectively. In general, granular materials having PS porosities closer to the densest possible packing, showed higher resilient modulus values.

Figure 3 Relationship between the volume of Secondary Structure and permanent deformation (PAPER I)
Figure 4 Relationship between PS porosity and Resilient Modulus (PAPER II)
6.0 CONCLUSIONS

A generalized granular material framework has been developed in this study. The model is based on packing theory. Unbound granular materials in base and subbase layers properties are required factors for the deformational behaviour of the whole pavement structure. As observed, in order to have good resistance to permanent deformation, the coarse aggregate particles should be interlocked well with the help of optimum amount of fines fraction. These can be attained by keeping the Disruption Potential value of the load carrying structure (PS) to be in an optimal range.

In addition, the influence of inter-particle contacts and pore spaces on resilient modulus was investigated. A decrease in PS porosity and an increase in coordination number for the nineteen unbound materials showed a significant increase of the resilient modulus. Materials with $n_{ps}$ smaller than 47.95 % and $c_{n_{ps}}$ greater than 6 (simple cubic coordination number) showed resilient modulus value of more than 400 MPa.

The study showed that the developed framework is a satisfactory and simple method to assess the risk against permanent deformation and load bearing capacity. It can also be concluded that the framework is capable of capturing experimentally observed characteristics of granular materials. It is also a promising step towards a better understanding of granular material behaviour.
References


Ching S. C., Anll M., and Sivanuja S. Sundarum, 1991. Properties of granular packings under low amplitude cyclic loading, *Department of Civil Engineering, University of Massachusetts, Amherst, MA 01003, USA*


APPENDED PAPERS
Packing theory-Based Framework to evaluate Permanent Deformation of Unbound Granular Materials

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ABSTRACT. Permanent deformation of unbound granular materials plays an essential role in the long term performance of a pavement structure. Stability of unbound granular materials is defined by the particle-to-particle contact of the system, the particle size distribution, and the packing arrangement. This paper presents a gradation model based on packing theory, to evaluate permanent deformation of unbound granular materials. The framework was evaluated by using ten unbound granular materials from different countries. The Disruption Potential (DP), which determines the ability of secondary structure to disrupt the primary structure, is introduced. This study also identified the amount of primary and secondary structures that may eventually be used as a design parameter for permanent deformation of unbound road layers. The evaluation of the model regarding permanent deformation behavior of granular materials is found to compare favourably with experimental results.

KEYWORDS: Gradation, Unbound Materials, Packing theory, Primary structure, Secondary structure, Disruption Potential, Permanent Deformation
1. Introduction

Unbound granular materials are widely used in road engineering practice. The structural integrity of a road is to a great extent defined by the performance of the base and subbase layers. Grain size distribution is a key characteristic of granular materials; better understanding of gradation is required in order to improve pavement design and construction procedures. Different researchers have examined the impact of grain size distribution on the performance of unbound materials (Thom and Brown, 1988; Dowson et al., 1996 and Kolisoja, 1998).

The effect of grading and compaction on plastic strain was also studied by Thom and Brown (1988), they concluded that the resistance to permanent strain decreased when the specimens were not compacted well for all grading types. Dowson et al. (1996) found that the resistance to plastic strain was higher for the densest grading so that the effect of grading on plastic strain is more considerable than the degree of compaction. Kolisoja (1998) argued that high permanent deformation is exhibited by unbound layers, which have high and low fines contents.

The diverse physical and geometrical properties of granular particles and their relative packing arrangement influence the mechanical behavior of granular materials (Santamarina, 2001).

Roque et al., (2006) developed a conceptual and theoretical approach to evaluate coarse aggregate structure based on gradation of asphalt mixtures. The approach provided a framework for gradation to evaluate the performance of asphalt mixtures. They defined the Dominant Aggregate Size Range (DASR) that forms the load carrying structural network of aggregates in asphalt mixtures. Porosity of the DASR was used as a criterion to distinguish between good and bad mixtures. They validated their framework by using an extensive range of laboratory and field asphalt mixtures; and the DASR porosity has satisfactorily identified mixtures with poor performance.

Guarin (2009) studied the effect of interstitial component (IC) which includes particle sizes smaller than the DASR and binder. He analysed the DASR disruption from two perspectives; local and global. He also introduced the Disruption Factor (DF), which is a parameter to evaluate the potential of IC particles to disrupt the DASR structure:

\[
DF = \frac{\text{Volume of potentially disruptive IC particles}}{\text{Volume of DASR voids}}
\]  

(1)
where, volume of potentially disruptive IC particles considers particles smaller than DASR and bigger than the voids of DASR; it is calculated by using volumetric relationships. Volume of DASR voids was determined as a function of the type of DASR structure (cubical or hexagonal) and the number of DASR particles. The type of packing arrangement (simple cubic or close hexagonal) was selected according to the DASR porosity.

The studies mentioned above examine the effect of gradation on the permanent deformation behaviour based strictly on materials with porosity between 26 % to 48 % and their theoretical DASR model was developed based on 2-D packing. The DASR approach was developed based on numerical packing analysis which is valid strictly for a fixed 2:1 size ratio between two contiguous sieve sizes. Furthermore, the DASR identification is performed graphically and can be somewhat subjective.

Lira et al. (2012) presented a 3-D packing performance evaluation framework for asphalt mixtures which removed the limitations mentioned above.

In the present paper the framework presented by Lira et al. (2012) is generalized further for performance evaluation of unbound granular materials, without the presence of bitumen. Furthermore, a new evaluation parameter, entitled “Disruption Potential (DP)” is introduced instead of the Disruption Factor (DF).

As the framework developed is based on packing considerations it is possible to generalize further introducing physical models for particle wear and degradation.

The framework model shows two basic components of the skeleton of unbound granular materials which are Primary Structure (PS) and Secondary Structure (SS). PS is a range of interactive grain sizes that forms the network of unbound granular materials and SS is a range of grain sizes smaller than the PS. The primary structure (PS) definition is an extension of the Dominant Aggregate Size Range (DASR).

The main objective of this paper is to present the Disruption Potential (DP), which is a modification of the DF, as a tool to evaluate the permanent deformation potential of unbound granular materials.

Ten unbound materials with known gradation and permanent deformation performance as reported by Ekblad (2007), Lekarp (1999), and Austin A. (2009) were used to evaluate the proposed framework. Ekblad (2007) used four similar crushed granite aggregates from
Skärlunda (Östergötland, Sweden) with different gradations (Granite 1, Granite 2, Granite 3, and Granite 4). Three Louisiana State base materials were taken from the second reference (Austin A., 2009); including limestone (LS), sandstone (SST) and granite (GR). The other three materials were obtained from Lekarp (1999), who evaluated typical Swedish base coarse materials including Granite (Gr), Sand and Gravel (S&G), and crushed concrete (CONC).

The DP of these ten unbound granular materials was analysed on the basis of permanent deformation results and compares favourably with experimental findings and provides a way to evaluate the performance of unbound granular materials.
2. Packing theory-based Framework Concept

2.1 Primary Structure

Unbound materials are composed of stones with a size distribution determined and described through a gradation analysis. In this paper, it is hypothesized that the skeleton of the materials must be composed of coarse aggregates supported by a limited amount of fine granular materials to effectively resist the deformation and distribute loads.

The interplay between the load carrying skeleton, the so called Primary Structure (PS), and the finer fraction (Secondary Structure (SS)) is shown in figure 1.

![Primary and Secondary Structures](image)

a) Low SS  b) Optimum SS  c) High SS (Disrupted)

**Figure 1 Primary and Secondary Structures**

One may observe that too low a percentage of fine materials may fail to provide an adequate support to PS (fig.1a), while too high a percentage of fine materials may result in PS particles loosing contact with each other (fig.1c), thus reducing load carrying capacity of the materials.

The particle to particle contact between aggregates composing the PS ensured is crucial for the good performance of the material. As it is known, cf. Lambe and Whitman (1965) the porosity of granular materials in the loose state is around 44-55%. This implies that the porosity of the load carrying PS should not be greater than 50%. It is not very common in aggregate gradations that a 45% concentration can be achieved by only one stone size.

The load carrying skeleton is thus composed of particles having a certain size range. The procedure to identify the size range of the PS (Similar framework has been recently presented by Lira et al. (2012) for asphalt mixtures) is presented for unbound granular materials below.
The developed analytical approach referred to the distribution of particles in terms of the sieve sizes \([D_i]\), (i.e., \(D_i\) is the diameter of sieves continuously from larger to smaller). The percentages by volume (concentrations) of two consecutive sieve sizes are denoted by \(\varphi_1\) and \(\varphi_2\). Thus, the mathematical expression of the weighted average grain size \([D_{w.\avg}]\) of the two contiguous sieve sizes can be calculated as follows:

\[
D_{w.\avg} = \frac{\varphi_1D_1 + \varphi_2D_2}{\varphi_1 + \varphi_2}
\] (2)

The skeleton of unbound granular materials was developed based on the 3-D densest possible packing (Cubic Close Packing, CCP) having a concentration determined as:

![Cubic Close Packing (CCP)](image)

**Figure 2. Cubic Close Packing (CCP)**

In Figure 2 the CCP structure has one-eighth of spheres at the eight corners and six hemispheres at the centres of six faces. Let ‘\(r\)’ be the radius of the sphere and ‘\(a\)’ be the length of the cubical structure. The diagonal of the face is \(4r\), so each side \(a\) is equal to \(2\sqrt{2}r\).

The volume of cubical structure is equal to \(a^3\)

\[
V = (2\sqrt{2}r)^3 = 16\sqrt{2}r^3
\] (3)

The total volume of spheres in the cubical structure is equal to:

\[
V_s = (8 \times \frac{1}{8} + 6 \times \frac{1}{3}) \times \frac{4\pi}{3} r^3 = \frac{16}{3}\pi r^3
\] (4)

Therefore, the concentration of the spheres in a CCP, i.e., the fraction of \(V_s\) over \(V\) becomes:

\[
\eta_{CCP} = \frac{16}{3} \frac{\pi r^3}{16\sqrt{2}r^3} = \frac{\pi}{3\sqrt{2}} = 0.74
\] (5)
Hexagonal close packing (HCP) and CCP have the same packing density of 0.74 and are known to be the densest possible packing of equal spheres.

This packing arrangement is supposed to be composed from two concentrations, the first concentration having a sieve size $D_1$ with a concentration of $\varphi_1 = 0.52$ (simple cubical). The second concentration is then be $\varphi_2 = 0.22$ with a sieve size of $D_2$.

In order to identify the upper limit of the PS, it is taken ratio of the two consecutive sieve sizes concentrations $[\varphi_1/\varphi_2 = 0.52/0.22 = 2.36]$ and their ratio of smaller to larger sieve sizes $(D_2/D_1)$. Substituting these two parameters in equation [2] gives equation [6b]:

$$D_{\text{w.avg}} = \frac{\varphi_1 + \frac{D_2}{D_1}}{\varphi_2 + 1} = \frac{2.36 + \frac{D_2}{D_1}}{2.36 + 1}$$  \hspace{2cm} (6a)$$

$$D_{\text{w.avg}} \leq \left[ 0.703 + 0.297 \frac{D_2}{D_1} \right] D_1$$  \hspace{2cm} (6b)$$

In order to consider the influence of disruptive materials (DM) on the stability of the skeleton, the limits are further defined in terms of weighted average void diameter $[d_{\text{w.avg}}]$.

$[d_{\text{w.avg}}]$ can be calculated using 3D simple cubic packing arrangement (Figure 3). $[d_{\text{w.avg}}]$ can be calculated using 3D simple cubic packing arrangement on the following expression:

$$2R_{\text{w.avg}} = D_{\text{w.avg}}$$

$$2R_{\text{w.avg}} = D_{\text{w.avg}}$$

Figure 3 Calculating the $d_{\text{w.avg}}$ from geometrical consideration
From figure 3 above the geometrical relationship is established as follows.

\[ d_{w.\text{avg}} = H - D_{w.\text{avg}} = \sqrt{3}D_{w.\text{avg}} - D_{w.\text{avg}} = 0.732D_{w.\text{avg}} \]  

(7)

This implies that if the size of void in the primary structure is greater than 0.732 times the sizes of each PS particles, it will be known that the particles are already disrupted and there would be no stone to stone contact between them.

Therefore, the upper limit of PS particles in terms of weighted average void diameter \( d_{w.\text{avg}} \) after a substitution of equation [7] into equation [6b] gives:

\[ d_{w.\text{avg}} \leq \left[ 0.515 + 0.217 \frac{D_2}{D_1} \right] D_1 \]  

(8)

Similarly, in order to show the lower limit of PS particles in terms of the diameter of two consecutive sieve sizes, the spacing between two particles was reviewed. The mathematical expression is shown in equation [9]. \( \varphi_{\text{max}} \) represents the maximum packing fraction (0.74) and \( \varphi \) is the volume fraction (Coussot, 2005).

\[ h \leq 2r \left[ \left( \frac{\varphi_{\text{max}}}{\varphi} \right)^{\frac{1}{3}} - 1 \right] \]  

(9)

where \( h \) represents the distance between particles and replaced as the smaller sieve size diameter \( (D_2) \) out of the two consecutive sieve sizes. And \( r \) denotes the radius of the larger sieve sizes \( (D_1/2) \).

\[ D_2 \leq 2 \left( \frac{D_1}{2} \right) \left[ \left( \frac{0.74}{\varphi} \right)^{\frac{1}{3}} - 1 \right] \]  

(10)

\[ \frac{D_2}{D_1} = \left[ \left( \frac{0.74}{\varphi} \right)^{\frac{1}{3}} - 1 \right] \]  

(11)
Table 1 shows the small sieve size standards according to the International ISO 565 (1987), American US.STD/ASTM E11 (1995), and British BS 410(1986).

**Table 1 Small sieve sizes standards**

<table>
<thead>
<tr>
<th>Sieve size standards</th>
<th>International ISO 565 (mm)</th>
<th>US.STD/ASTM E11 (Mesh)</th>
<th>BSS 410 (Mesh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.75</td>
<td>4</td>
<td>3½</td>
<td></td>
</tr>
<tr>
<td>4.00</td>
<td>5</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3.35</td>
<td>6</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>2.80</td>
<td>7</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>2.36</td>
<td>8</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td>10</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

For instance, in Table 1 the commonplace for mesh number 8 and 4 of two consecutive smaller/larger sieve sizes in European and US specification are 2/4 mm and 2.36/4.75 mm respectively. Thus, these common two consecutive smaller to larger sieve sizes in unbound granular materials give minimum ratio approximately 0.5.

In order to find the lower limit of PS particles, it is taken the minimum ratio of two consecutive smaller to larger sieve sizes \( \frac{D_2}{D_1} = 0.5 \) to substitute in equation [11]. The resulting minimum concentration or volume of fraction becomes \( \varphi = 0.22 \). Therefore, having a maximum concentration \( \varphi_{\text{max}} = 0.74 \) leads the concentration of the two consecutive larger \( D_1 \) and smaller \( D_2 \) sieve sizes to have \( \varphi_1 = 0.22 \) and \( \varphi_2 = 0.52 \) concentrations respectively.

Using equation [6a] the lower limit of the weighted average sieve sizes can also be calculated. The ratio of the two consecutive sieve sizes concentrations then be \( \varphi_1/\varphi_2 = 0.22/0.52 = 0.423 \).

\[
D_{w,\text{avg}} = \frac{\varphi_1 + \frac{D_2}{D_1}}{\varphi_2 + 1} \quad \frac{0.423 + \frac{D_2}{D_1}}{0.423 + 1} \quad (12a)
\]

\[
D_{w,\text{avg}} \geq \left[ 0.297 + 0.703 \frac{D_2}{D_1} \right] D_1 \quad (12b)
\]
Expressing in terms of the weighted average void diameter \(d_{w,\text{avg}}\) based on Equation (7) gives:

\[
d_{w,\text{avg}} \geq \left[ 0.217 + 0.515 \frac{D_2}{D_1} \right] D_1 \tag{13}
\]

Finally, Equation (14) is combined from Equations (8) and (13) to put together the upper and lower limits of \(d_{w,\text{avg}}\) to identify PS ranges of unbound granular materials based on their packing arrangement. This analysis goes down until sieve size of 2.00 mm and 2.36 mm of European and American sieve size standard specifications respectively.

\[
\left[ 0.217 + 0.515 \frac{D_2}{D_1} \right] D_1 \leq d_{w,\text{avg}} \leq \left[ 0.515 + 0.217 \frac{D_2}{D_1} \right] D_1 \tag{14}
\]
3. Definition of Disruption Potential for evaluating the Permanent Deformation Behaviour of Unbound Granular Materials

The Disruption Potential (DP) is defined as the ratio of the volume of potentially disruptive fine material over the free (available) volume within the primary structure. DP can be obtained for any material regardless of its porosity. The mathematical expression for DP is illustrated in the following equation [15].

$$ DP = \frac{V_{DM}^{SS}}{V_{free}^{PS}} $$

(15)

where, $V_{DM}^{SS}$ = the volume of disruptive material

$V_{free}^{PS}$ = the free volume within the primary structure

The volumetric calculations in the analysis assumed constant dry density and specific gravity of aggregates throughout each sieve size. The disruptive materials volume ($V_{DM}^{SS}$), includes SS particles larger than 0.225 times the smallest grain size of the PS; 0.225D grain size corresponds to the average void size of spherical particle system in the densest possible packing arrangements (Cubic and Hexagonal close packing). In addition, the Bailey method also used this factor to establish control sieves to separate coarse and fine aggregates (Vavrik et al., 2002). With this assumption for materials having a PS range from $L_1$ to $L_2$, the Disruptive Materials (DM) ranges from $L_2$ to 0.225$L_2$ (i.e., $L_1$ and $L_2$ is the upper and lower limit of PS respectively). Table 2 illustrates the lower limits of DM for different smallest PS particles size.

Table 2 Proposed Lower Limits of Disruptive Material (DM)

<table>
<thead>
<tr>
<th>Smallest PS particle size [mm]</th>
<th>0.225*Smallest PS sieve size,[mm]</th>
<th>Lower Limit of DM sieve,[mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1.76</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>0.88</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.44</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>0.22</td>
<td>0.25</td>
</tr>
</tbody>
</table>
4. Framework Validation

In order to validate the proposed framework, gradations data were collected from three different references (Ekblad, 2007; Lekarp, 1999; and Austin. 2002). The unit volume of aggregates and voids should be calculated as a function of the specific gravity ($G_s$), dry density ($\rho_d$), and water content ($\omega$) of the aggregates. Constant dry density was assumed throughout each grain size.

Ekblad (2007) used one aggregate material type from Skärlunda (Östergötland Sweden) with four different gradations (Granite 1, Granite 2, Granite 3, and Granite 4). These are typical base materials of foliated medium grained granite with quartz, and plagioclase as main constituents. The maximum particle size of the granites was 90 mm; and the particle size distributions are shown in Figure 4. Specific gravity, water content, and unit volume of aggregates and voids are shown in Table 3.

Table 3 Physical properties of the four granite materials (Ekblad. 2007)

<table>
<thead>
<tr>
<th>Materials</th>
<th>Granite 1</th>
<th>Granite 2</th>
<th>Granite 3</th>
<th>Granite 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry density, g/cm$^3$</td>
<td>1.923</td>
<td>2.112</td>
<td>2.154</td>
<td>2.134</td>
</tr>
<tr>
<td>Water content, [% by weight]</td>
<td>0.500</td>
<td>4.000</td>
<td>1.500</td>
<td>2.000</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>2.637</td>
<td>2.630</td>
<td>2.629</td>
<td>2.615</td>
</tr>
<tr>
<td>Unit volume of aggregates, cm$^3$</td>
<td>0.729</td>
<td>0.803</td>
<td>0.819</td>
<td>0.816</td>
</tr>
<tr>
<td>Unit volume of voids, cm$^3$</td>
<td>0.271</td>
<td>0.197</td>
<td>0.181</td>
<td>0.184</td>
</tr>
</tbody>
</table>

Figure 4 Grain size distributions for the four granite materials (Ekblad. 2007)
Table 4 shows the physical properties of materials obtained from Lekarp (1999); he studied three Swedish materials including Crushed Granite (GR), Sand and Gravel (S&G), and Crushed concrete (CONC). The particles size distributions of the materials are plotted in Figure 5. The first two materials (GR, S&G) are used mainly as subbase materials in Sweden. The crushed concrete (CONC) is a recycled building material included in the study because of the growing need in road construction industry to utilize alternative materials (recycled or by-products).

**Table 4 Physical properties of the three materials (GR, LS, S&G and CONC) (Lekarp, 1999)**

<table>
<thead>
<tr>
<th>Materials</th>
<th>S&amp;G</th>
<th>GR</th>
<th>CONC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry density, g/cm³</td>
<td>2.07</td>
<td>2.2</td>
<td>1.93</td>
</tr>
<tr>
<td>Water content, [% by weight]</td>
<td>2.6</td>
<td>2</td>
<td>4.4</td>
</tr>
<tr>
<td>Saturation, S, [%]</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>specific gravity</td>
<td>2.274</td>
<td>2.374</td>
<td>2.248</td>
</tr>
<tr>
<td>Unit volume of aggregates, cm³</td>
<td>0.91</td>
<td>0.927</td>
<td>0.858</td>
</tr>
<tr>
<td>Unit volume of voids, cm³</td>
<td>0.09</td>
<td>0.073</td>
<td>0.142</td>
</tr>
</tbody>
</table>

**Figure 5 Grain size distributions for the three granular materials (Lekarp, 1999)**

The three Louisiana State base materials including limestone (LS), sandstone (SST) and granite (GR) were studied by Austin, A (2009); figure 6 shows their particle size distribution. All materials have maximum nominal aggregate size of 25 mm. Some physical properties of the materials are depicted in Table 5.
Table 5  *Physical properties of the three Louisiana State materials* (Austin. 2002)

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>LS</th>
<th>SST</th>
<th>Gr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Gravity</td>
<td>2.708</td>
<td>2.642</td>
<td>2.671</td>
</tr>
<tr>
<td>Max. dry density in g/cm³</td>
<td>2.27</td>
<td>2.18</td>
<td>2.11</td>
</tr>
<tr>
<td>Unit Volume of aggregates, cm³</td>
<td>0.84</td>
<td>0.826</td>
<td>0.792</td>
</tr>
<tr>
<td>Unit Volume of voids, cm³</td>
<td>0.16</td>
<td>0.174</td>
<td>0.208</td>
</tr>
<tr>
<td>Optimum Moisture Content, %</td>
<td>6.5</td>
<td>7.1</td>
<td>6</td>
</tr>
<tr>
<td>Degree of saturation, %</td>
<td>80.7</td>
<td>88</td>
<td>76.3</td>
</tr>
<tr>
<td>AASHTO Classification</td>
<td>A-1-b</td>
<td>A-1-b</td>
<td>A-1-b</td>
</tr>
<tr>
<td>USCS Classification</td>
<td>GW/Sand</td>
<td>GW/Sand</td>
<td>GW/Sand</td>
</tr>
<tr>
<td>Coarse aggregate angularity (%)</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 6  *Grain size distributions for the three granular materials* (Austin. 2002)
5. Results and Discussions

Table 6 shows the volumetric composition of the ten granular materials describing volumes of primary structure ($V_{ps}$) and secondary structure ($V_{ss}$) as well as the portion of aggregates greater than the primary structure $V_{agg (>PS)}$.

Table 6 Volumetric Composition of the materials

<table>
<thead>
<tr>
<th>Materials</th>
<th>Volumetric Material Distributions</th>
<th>Aggregate composition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_T$ (%)</td>
<td>$V_{agg}$ (%)</td>
</tr>
<tr>
<td>Granite 1</td>
<td>100</td>
<td>72.9</td>
</tr>
<tr>
<td>Granite 2</td>
<td>100</td>
<td>80.3</td>
</tr>
<tr>
<td>Granite 3</td>
<td>100</td>
<td>81.93</td>
</tr>
<tr>
<td>Granite 4</td>
<td>100</td>
<td>81.6</td>
</tr>
<tr>
<td>S&amp;G</td>
<td>100</td>
<td>91.02</td>
</tr>
<tr>
<td>GR</td>
<td>100</td>
<td>92.68</td>
</tr>
<tr>
<td>CONC</td>
<td>100</td>
<td>85.85</td>
</tr>
<tr>
<td>LS</td>
<td>100</td>
<td>84</td>
</tr>
<tr>
<td>SST</td>
<td>100</td>
<td>82.6</td>
</tr>
<tr>
<td>Gr</td>
<td>100</td>
<td>79.2</td>
</tr>
</tbody>
</table>

Table 7 The permanent, resilient, and normalized permanent strains of the materials

<table>
<thead>
<tr>
<th>Materials</th>
<th>$\varepsilon_p$ [µε]</th>
<th>$\varepsilon_r$ [µε]</th>
<th>$\varepsilon_p/\varepsilon_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite_1</td>
<td>12410</td>
<td>283</td>
<td>43.85</td>
</tr>
<tr>
<td>Granite_2</td>
<td>5560</td>
<td>326</td>
<td>17.06</td>
</tr>
<tr>
<td>Granite_3</td>
<td>5010</td>
<td>300</td>
<td>16.7</td>
</tr>
<tr>
<td>Granite_4</td>
<td>3710</td>
<td>319</td>
<td>11.63</td>
</tr>
<tr>
<td>S&amp;G</td>
<td>27000</td>
<td>550</td>
<td>49.10</td>
</tr>
<tr>
<td>GR</td>
<td>11000</td>
<td>420</td>
<td>26.20</td>
</tr>
<tr>
<td>CONC</td>
<td>5400</td>
<td>390</td>
<td>13.85</td>
</tr>
<tr>
<td>LS</td>
<td>19800</td>
<td>750</td>
<td>26.4</td>
</tr>
<tr>
<td>SST</td>
<td>38000</td>
<td>1650</td>
<td>23.03</td>
</tr>
<tr>
<td>Gr</td>
<td>14000</td>
<td>1000</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 7 presents the permanent and resilient strains, and normalized permanent strains that obtained from the repeated loading triaxial test (RLTT) measurement system (Ekblad, 2007; Lekarp, 1999; Austin, 2002). European standard (prEN 13286-7) allows a maximum confining stress up to 110 kPa for conditioned sample in triaxial testing (method A). The resilient strains for Granite 1 to 4 were back calculated using their resilient modulus value with known mean normal stress of 150 kPa and confining stress of 100 kPa that results a
deviatoric stress of 150 kPa (Ekblad, 2007). Similarly, for S&G, GR and CONC with a deviatoric stress of 150 kPa, the resilient strains were obtained with known value of resilient modulus (Lekarp, 1999). Both permanent strain and resilient strain values for LS, SST and Gr materials were found after 4000 load cycle (Austin, 2009). In order to make all values of permanent strains consistent for comparison purpose, normalizations were made using their own resilient strains for each material.

Table 8 Primary Structure and Disruptive Material ranges

<table>
<thead>
<tr>
<th>Materials</th>
<th>PS, Range (mm)</th>
<th>DM, Range (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite_1</td>
<td>63.0-4.0</td>
<td>4.0-1.0</td>
</tr>
<tr>
<td>Granite_2</td>
<td>63.0-2.0</td>
<td>2.0-0.5</td>
</tr>
<tr>
<td>Granite_3</td>
<td>63.0-2.0</td>
<td>2.0-0.5</td>
</tr>
<tr>
<td>Granite_4</td>
<td>63.0-2.0</td>
<td>2.0-0.5</td>
</tr>
<tr>
<td>S&amp;G</td>
<td>63.0-4.0</td>
<td>4.0-1.0</td>
</tr>
<tr>
<td>GR</td>
<td>63.0-2.0</td>
<td>2.0-0.5</td>
</tr>
<tr>
<td>CONC</td>
<td>31.5-2.0</td>
<td>2.0-0.5</td>
</tr>
<tr>
<td>LS</td>
<td>9.5-2.36</td>
<td>2.36-0.6</td>
</tr>
<tr>
<td>SST</td>
<td>9.5-2.36</td>
<td>2.36-0.6</td>
</tr>
<tr>
<td>Gr</td>
<td>9.5-2.36</td>
<td>2.36-0.6</td>
</tr>
</tbody>
</table>

In Table 8, it is shown the range of primary structure, and Disruptive Material (DM) ranges. The LS and SST showed PS ranges from 9.5 to 2.36 mm; they comprised relatively small ranges and less volume of PS; exhibiting unstable condition. This confirms that unbound materials should be coarse enough aggregates to ensure more stable structure. Granular materials that showed large proportion of PS (>50%) performed well in terms of stability.

5.1 Disruption Potential (DP)

As discussed in section 3, DP can be determined from equation [15] by knowing the percent volume of disruptive materials and the free volume available in the SS and PS respectively. The percent volume of aggregate in the disruptive material (DM) ranges, the free volume within the PS, and DP values are shown in Table 9.
Table 9 Disruption Potential Calculation

<table>
<thead>
<tr>
<th>Materials</th>
<th>$V_{SSDM}^{%}$</th>
<th>$V_{PS}^{%}$ free</th>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite_1</td>
<td>2.3328</td>
<td>25.26</td>
<td>0.092</td>
</tr>
<tr>
<td>Granite_2</td>
<td>6.0225</td>
<td>16.74</td>
<td>0.36</td>
</tr>
<tr>
<td>Granite_3</td>
<td>7.371</td>
<td>14.09</td>
<td>0.523</td>
</tr>
<tr>
<td>Granite_4</td>
<td>8.976</td>
<td>12.51</td>
<td>0.718</td>
</tr>
<tr>
<td>S&amp;G</td>
<td>35.49</td>
<td>4.14</td>
<td>8.572</td>
</tr>
<tr>
<td>GR</td>
<td>2.78</td>
<td>6.34</td>
<td>0.438</td>
</tr>
<tr>
<td>CONC</td>
<td>9.45</td>
<td>11.01</td>
<td>0.858</td>
</tr>
<tr>
<td>LS</td>
<td>14.28</td>
<td>4.88</td>
<td>2.926</td>
</tr>
<tr>
<td>SST</td>
<td>8.673</td>
<td>5.742</td>
<td>1.51</td>
</tr>
<tr>
<td>Gr</td>
<td>8.316</td>
<td>8.84</td>
<td>0.941</td>
</tr>
</tbody>
</table>

DP value can be helpful to know the potential of the materials resistance to permanent deformation. Accordingly, the Swedish materials exhibited relatively good potential in resistance to permanent deformation except Granite_1 and S&G.

Figure 7 Relationship between the disruption potential and permanent deformation

Figure 7, showed the relationship between normalized permanent strains versus DP values. Low resistances to permanent deformation were exhibited by granular materials
having very low and high volume of potentially disruptive materials (Granite_1 and S&G). This could be explained by theory that if SS is not fully fit into the voids between the PS, it will therefore be exposed to high deformation during external loads. However, granular materials having sufficient SS (Granite_2, 3, 4, GR, CONC, SST and Gr) that supported the PS effectively showed high resistance to permanent deformation.

5.2 Effect of Secondary structure on Permanent deformation

According to hypothesis, figure 8 shows that low and high percent volume of SS has a great effect on the deformation behaviour of unbound materials. As the amount of SS increases and decreases the material shows poor performance in terms of permanent deformation. Good performance is shown on materials with optimum amount of SS that can support the PS in the overall skeleton of granular materials.

![Figure 8 Relationship between the volume of Secondary structure and permanent deformation](image)

5.3 Comparison to Disruption Factor-Based Framework

This Disruption Factor (DF) can be calculated by using eq. [1] (Guarin, 2009). Table 10a shows DF values for close hexagonal structure (Octahedral and tetrahedral). The permanent deformation behaviour exhibited poor performance with low value of DF.
Table 10a The Disruption Factor for Close Hexagonal packing materials

<table>
<thead>
<tr>
<th>Materials</th>
<th>Porosity of PS %</th>
<th>DF Octahedral (DF_o)</th>
<th>DF Tetrahedral (DF_t)</th>
<th>εp/εr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite_1</td>
<td>31.98</td>
<td>1.24</td>
<td>3.65</td>
<td>43.85</td>
</tr>
<tr>
<td>Granite_2</td>
<td>31.74</td>
<td>4.08</td>
<td>8.52</td>
<td>17.06</td>
</tr>
<tr>
<td>Granite_3</td>
<td>36.1</td>
<td>4.86</td>
<td>9.62</td>
<td>16.7</td>
</tr>
<tr>
<td>CONC</td>
<td>32.46</td>
<td>4.47</td>
<td>11.6</td>
<td>13.85</td>
</tr>
</tbody>
</table>

Table 10b presents DF for cubical arrangement; DF seems to be in the optimal DF ranges and exhibited comparatively good resistance to permanent deformation. Guarin (2009) also found good performance for asphalt mixture having DF values less than one and suggested DF value as a reference, for cubical DASR structure, to be between 0.65 and 0.85.

Table 10b The Disruption Factor for Simple cubic packing materials

<table>
<thead>
<tr>
<th>Materials</th>
<th>PS Range in mm</th>
<th>Porosity of PS</th>
<th>PS Packing</th>
<th>DF Cubical</th>
<th>εp/εr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite_4</td>
<td>63-2.0</td>
<td>44.51</td>
<td>Cubical</td>
<td>0.97</td>
<td>11.63</td>
</tr>
<tr>
<td>Gr</td>
<td>9.5-2.36</td>
<td>53.17</td>
<td>Cubical</td>
<td>0.56</td>
<td>14</td>
</tr>
</tbody>
</table>

As mentioned before, DF considers two types of voids for hexagonal close packing structure (tetrahedral and octahedral); therefore two DF values (DF_o and DF_t) was calculated (Table 9a). Since DP includes the total air voids from PS; then for comparison purpose a single DF value for hexagonal structure can be calculated by considering the total sum of the disruptive materials and voids in octahedral and tetrahedral structure with the following equation [16].

\[
DF_T = \frac{V_{PDM,o} + V_{PDM,t}}{V_{v,PS,o} + V_{v,PS,t}}
\]

where, \(V_{PDM,o}\) and \(V_{PDM,t}\) are the volume of disruptive materials for octahedral and tetrahedral structure respectively. \(V_{v,PS,o}\) and \(V_{v,PS,t}\) are the volumes of voids in PS for octahedral and tetrahedral structure respectively. Table 11 shows the total disruption factor (DF_T) resulted from both octahedral and tetrahedral voids. The four materials showed close hexagonal arrangement according to their PS porosities.
Table 11 *The Total Disruption Factor Calculations*

<table>
<thead>
<tr>
<th>Materials</th>
<th>$V_{PDM,o} + V_{PDM,t}$</th>
<th>$V_{r,PS,o} + V_{r,PS,t}$</th>
<th>$DF_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite 1</td>
<td>2.3</td>
<td>1.3</td>
<td>1.77</td>
</tr>
<tr>
<td>Granite 2</td>
<td>2.3</td>
<td>0.444</td>
<td>5.18</td>
</tr>
<tr>
<td>Granite 3</td>
<td>7.4</td>
<td>1.225</td>
<td>6.04</td>
</tr>
<tr>
<td>CONC</td>
<td>9.5</td>
<td>1.523</td>
<td>6.24</td>
</tr>
</tbody>
</table>

DF_T and DP results are shown in Figure 9. The results are obviously not similar, but the trend for increasing the DF_T and DP exhibited good resistance to permanent deformation for granular materials having PS porosities less than 50%. Particularly, for the four close hexagonal granular materials, the DP values showed less than one.

![Figure 9](image_url)

*Figure 9 Total Disruption Factor (DF_T) and Disruption Potential (DP)*
6. Conclusions

In the previous study DF was calculated based on the porosity of DASR, type of DASR structure (cubical or hexagonal), and the number of DASR particles. However, in this study, DP can be calculated regardless the porosity of PS and for all ranges of PS porosities getting measured weight and density of materials.

The permanent deformation behaviour of unbound granular materials has been evaluated and shows the effect of SS on the composition of granular materials is considerably high. In the framework the DP model regarding permanent deformation behaviour of granular materials is found to compare favourably with experimental results.

Most stable unbound granular materials showed DP which ranged from 0.5 to 0.9. These materials satisfactorily exhibited best performance in terms of resistance to permanent deformation. Therefore, as a reference optimal unbound granular material composition should have DP between 0.5 and 0.9.
References


Packing Theory-Based Framework for Evaluating the Resilient Modulus of Unbound Granular Materials

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ABSTRACT. Unbound granular material is the largest material used in highway construction and its properties are important for the structural quality of the pavement. The objective of this paper is to investigate the influence of packing theory-based parameters on the resilient modulus of unbound granular materials. In this study nineteen differently graded unbound granular materials were used from two different countries (USA and Sweden). The study identified the calculation of load carrying aggregates porosity and their contact points per particle (coordination number) as key parameters for evaluating the resilient modulus of unbound granular materials. It was found that higher resilient modulus is associated with lower porosity of load carrying particles and higher number of contact points per particle. The experimental results agree well with the framework predictions.

KEYWORDS: Unbound materials, packing theory, primary structure, porosity, coordination number, and resilient modulus
1. Introduction

In flexible pavements, unbound base layer plays a major role in spreading the axle loads imposed at the surface so that the stresses transmitted to the subbase and subgrade do not exceed the strength of this layer. The unbound base layer therefore must possess high stiffness and strength. Among several characteristics, gradation, porosity, and number of contact points between aggregates in a granular mass can largely influence the bearing capacity of the overall structure. The characteristics of the constituent aggregate materials are also important in evaluating their strength and stiffness properties.

The resilient modulus of unbound granular materials has been studied experimentally by many researchers on the basis of responses from static and dynamic loading (e.g., Trollope et al., 1962; Thom, N and Brown, S., 1988; Lekarp, 1999; Ekblad, 2007). The effect of grain size distribution, porosity and contact points in unbound aggregate materials on resilient modulus properties is still not very well understood. A few researchers had attempted to link some material properties with gradation characteristics including the relationship between the average contact points per particle and porosity as well as void ratio (e.g. Oda, M. 1977; Suzuki, M., 1981). However, all these investigations applied a uniform analysis to the whole aggregate size range assuming that all aggregates contribute equally to the load carrying characteristics of the materials and not accounting for the different roles of different aggregate sizes.

Yideti, et al. (2012) presented a packing theory-based framework developed for identifying part of the gradation curve which provides a major load carrying structure in unbound granular materials. In this paper such framework was used to examine the influence of the PS porosity and coordination number on the resilient modulus of nineteen different geologically formed unbound granular materials (Ekblad, 2007; Lekarp, 1999; Austin, 2009, and David et. al. 2009). Section 2 presents a literature review of the influence of density, gradation, porosity, and coordination number of granular materials on resilient modulus properties. Section 3 presents the theoretical framework used to calculate PS porosity and coordination number. Section 4 reviews the physical behaviour of the nineteen unbound base and subbase materials. The framework evaluation is presented in section 5, illustrating the effects of PS porosity and coordination number on resilient modulus. In conclusion, the results presented show that the PS porosity and coordination number are related to the resilient modulus values of unbound granular materials.
2. Literature review

Unbound pavement layers can be characterized based on their physical properties such as gradation, hardness, durability, and shear strength. Trollope et al. (1962) conducted repeated load triaxial tests on uniform sand and found that the resilient modulus increased by up to 50% when changed from loose to dense condition. Robinsson (1974) also observed the number of particle-to-particle contacts increases strongly with increased density, resulting from additional compaction of the particulate system. Lambe and Whitman (1979) stated that the looser the soil the smaller the modulus for a given loading increment and leads to lack of interlocking which results in a decreasing volume (i.e., contraction) when its shape changed by shearing. They also stated that densest possible packing for unbound granular materials dilates during shear.

Particle size distribution of granular materials affects important properties of unbound granular material such as resilient modulus, permanent deformation, durability, and permeability (Thom, N and Brown, S., 1988). Density and packing structure of compacted material significantly affect resilient modulus. Lekarp et al. (2000) also suggested that increasing the density of granular materials significantly changes their response under static loading, resulting in stiffer.

The selection of the optimal grain size distribution for unbound pavement layers is normally accomplished by meeting broad gradation specifications which may reflect the characteristics of locally available aggregate sources, typical traffic loading and the local climate. Power law parameters were developed by Birgisson and Ruth (2001) to evaluate and classify grain size distribution for aggregates in asphalt mixtures based on their performance. In addition, Ruth et al. (2002) presented a power law-based approach for the selection of aggregate gradations.

2.1 Porosity

In geotechnical engineering, porosity is the parameter describing the amount of open space in granular materials; it is the ratio of the volume of voids ($V_v$) over the total volume ($V_T$) (Equation (1)).

$$\text{Porosity, } n = \frac{V_v}{V_T} = \frac{V_v}{V_T}$$  \hspace{1cm} (1)
The porosity of a stable mass of identical cohesionless spheres depends on the way in which the spheres are arranged. For instance, in rhombohedral and simple cubic packing configurations, the porosity is close to 26% and 48% respectively. Such ideal arrangements however not often realized in practice (Smith et al. 1929).

In Table 1, the theoretical porosities of spherical particles are shown (d_p is the particle diameter). Additionally, the number of contact points per particle and their unit cell volume for the four major packing arrangements (Cubic, Orthorhombic, Tetrahedral and Rhombohedral) are tabulated.

**Table 1 Characteristics of Packing of Uniform Spheres**

<table>
<thead>
<tr>
<th>Packing type</th>
<th>Number of Contact points per particle</th>
<th>Volume of Unit cell</th>
<th>Porosity</th>
<th>Packing factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cubic</td>
<td>6</td>
<td>d_p^3</td>
<td>47.64</td>
<td>1</td>
</tr>
<tr>
<td>Orthorhombic</td>
<td>8</td>
<td>√3/2d_p^3</td>
<td>39.54</td>
<td>√3/2</td>
</tr>
<tr>
<td>Tetrahedral</td>
<td>10</td>
<td>0.75d_p^3</td>
<td>30.19</td>
<td>0.75</td>
</tr>
<tr>
<td>Rhombohedral</td>
<td>12</td>
<td>1/√2d_p^3</td>
<td>25.95</td>
<td>1/√2</td>
</tr>
</tbody>
</table>

Stone to stone contact is crucial for adequate strength and density of unbound granular materials. The porosity of granular materials also varies based on the shape of the grains, the uniformity of the grain size, the type of granular materials, and the condition of particle to particle interlocking.

Roque et al. (2006) studied a new gradation-based framework for identifying and assessing the coarse aggregate structure of dense-graded asphalt mixtures for resistance to rutting. They examined the porosity of the Dominant Aggregate Size Range (η_DASR) to evaluate coarse aggregate structure of asphalt mixtures. It was found that coarse aggregates within an asphalt mixture should have a maximum porosity of 50% for the particles to be in contact with each other.
2.2 Coordination number

Smith, et al. (1929) described the average number of contacts per sphere as a function of porosity. They experimentally evaluated a relationship between the coordination number (cn) and the porosity (n) in random assemblies of spherical particles. The relationship was developed based on the two systematic arrangements (close-hexagonal and simple cubical). The proposed relationship is shown below in Equation (2).

\[ cn = 26.4858 - \frac{10.7262}{1-n} \]  

(2)

Table 2 shows four different packing arrangements on equal sizes of spheres in which their coordination number was determined by the void ratio (e).

**Table 2** Four systematic arrangements of homogeneous spheres (Graton and Fraser, 1935)

<table>
<thead>
<tr>
<th>Packing arrangement</th>
<th>Void ratio e</th>
<th>Co-ordination number (cn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cubic</td>
<td>0.910</td>
<td>6</td>
</tr>
<tr>
<td>Orthorhombic</td>
<td>0.654</td>
<td>8</td>
</tr>
<tr>
<td>Tetrahedral</td>
<td>0.433</td>
<td>10</td>
</tr>
<tr>
<td>Rhombohedral</td>
<td>0.350</td>
<td>12</td>
</tr>
</tbody>
</table>

Field (1963) developed an equation to find the coordination number for materials having different grain sizes with a rounded shape of particles. It is a function of the void ratio (e) of granular materials. The mathematical expression is mentioned below in Equation (3).

\[ cn = \frac{12}{1+e} \]  

(3)

Gray (1968) proposed an empirical relationship between the coordination number (cn) and porosity (n) in Equation (4).

\[ cn = \frac{3.1}{n} \]  

(4)

Ching et al. (1991) also presented a relationship between the void ratio and the coordination number for the packing of different spheres. They stated that the coordination
number should be a function of a limited range of void ratios. The calculation of coordination number (cn) is shown for the range of void ratios (e) between 0.35-0.85 in Equation (5).

\[ cn = 8*(1.66 - e) \]  \hspace{1cm} (5)

3. Theoretical Framework

The packing theory-based framework presented in Yideti et al. (2012) identifies two basic components of the skeleton of unbound granular materials which are Primary Structure (PS) and Secondary Structure (SS). PS is a range of interactive grain sizes that forms the load carrying network of unbound granular materials. SS is a range of grain sizes smaller than the PS and fills the gaps between the PS.

The framework is built on the hypothesis that the unbound granular materials consist of the load carrying skeleton, the so called Primary Structure (PS), and the finer fraction (Secondary Structure, SS), which resides in-between the primary structure particles and provides stability. The particle-to-particle contact between aggregates and their porosity in the PS is crucial for the stiffness and strength properties of the materials. As it is known, cf. Lambe and Whitman (1979) the porosity of granular materials in their loosest state is around 44-55%. This implies that the porosity of the load carrying PS approximately should not be greater than 50 %, in order to ensure the contact between aggregates in PS. However, it is not very common in aggregate gradations that a 45 % concentration can be achieved by only one stone size. The procedure to identify the range of Primary Structure is presented in Yideti, et al. (2012).

In this paper, the calculation of PS porosity is briefly reviewed and a revised coordination number relationship is presented and used to evaluate the resilient modulus property of unbound granular materials. The calculations of PS porosity (\(n_{ps}\)) and coordination number (\(c_{n_{ps}}\)) are expressed in the following section.

3.1 Primary Structure Porosity (\(n_{ps}\))

Primary Structure porosity (\(n_{ps}\)) in unbound granular materials was defined by Yideti et al (2012) as the fraction of the volume of voids in the PS over the total volume of granular mix (skeleton). The volume of voids in PS is everything in the skeleton that is not considered to be part of the PS, and the total volume of the granular mix (skeleton) is all except the volume of particles bigger than the Primary Structure.
For instance, let be PS range is from \( L_1 \) to \( L_2 \), where, \( L_1 \) and \( L_2 \) are the upper and lower gradation size limits of PS respectively. The mathematical expression is shown in Equation (6).

\[
n_{PS} = \frac{V^{PS}_v}{V^{PS}_T} = \frac{V^{SS}_v + V_v}{V_T - V_{AGG(\geq L_1)}}
\]

where:

\( V_T \) : Volume of total granular mix.

\( V_{agg(\geq L_1)} \) : Volume of granular materials retained on \( L_1 \) sieve

\( V^{SS}_v \) : Volume of Secondary Structure (granular materials smaller than PS)

\( V_v \) : Volume of voids in granular mix

In unbound granular materials the volume of voids in PS \( (V^{PS}_v) \) comprises the volume of SS and the volume of voids in the granular mix \( (V_v) \) that are calculated by volumetric relationships from having known dry density and specific gravity of the material.

### 3.2 Primary Structure Coordination number \((cn_{PS})\)

Although, the stability of granular materials varies based on the shape of the grains, the uniformity of the grain size, and the type of granular materials; the amount of particle to particle contact has a major impact on its stability. In the present paper, the influence of the configuration of stone contact on the load carrying portion of the granular material (Primary Structure) is also presented.

In this framework, a relationship between PS porosity \((n_{ps})\) and PS coordination number \((cn_{ps})\) was developed by fitting the theoretical number of contact points per particle to their corresponding porosities. The theoretical porosities and their corresponding coordination numbers for materials having porosity between 25.95 % and 47.64 % are presented in Table 1. The developed mathematical relationship and the fitting curve are shown in Equation (7) and Figure (1) respectively.

\[
cn_{PS} = 2.827 * \left[ \frac{n_{ps}}{100} \right]^{-1.069}
\]

where, \( n_{ps} \) is the PS porosity in percent.
Figure 1  Relationship between (cn) and (n) for theoretical porosities

4. Materials

The physical and mechanical properties of nineteen unbound granular materials were collected from four different sources (Ekblad, 2007; Lekarp, 1999; Austin, 2009, and David et al. 2009). They are all typical base and sub-base materials of different stone types and mineral structure such as granite, dolomite, limestone and sandstone. The maximum grain size for all the Swedish materials (Granite_1, Granite_2, Granite_3, Granite_4 S&G, and CONC) was 90 mm. The three Louisiana state base materials (LS, SST, and Gr) and the ten Missouri materials (Table 3a and 3b) had maximum sieve sizes of 37.5 mm and 25 mm respectively.

The materials are grouped according to their sources. The gradations for the four granite materials which originated from Skärlunda (Östergötland Sweden) are plotted in Figure 2. Granite 1 and 2 depicted above the maximum density line (MDL), and Granite 3 and 4 are clearly below the MDL.

The gradation of the remaining two Swedish materials including Sand and Gravel (S&G) and Crushed Concrete (CONC) is also plotted in Figure 3.
Figure 2 Grain size distributions for the four granite materials (Ekblad, 2007)

Figure 3 Grain size distributions for the three granular materials (Lekarp, 1999)
The three Louisiana state base materials are also grouped according to the Unified Soil Classification System (USCS) using their physical properties (Austin. 2009). They all are well-graded (GW) and their gradation curves are plotted in Figure 4.

![Figure 4](image)

**Figure 4** Grain size distributions for the three granular materials (Austin. 2009).

Ten unbound granular base materials from Missouri with various geological formations were used by Missouri University of S&T to determine resilient modulus (David et al., 2009). Five of the materials were in the As-Delivered gradation condition and designated A-D and the remaining five materials were composed by adding fines (minus 0.075 mm sieve) to the A-D materials. These additional five materials are designated as W-F (With-Fines). The nominal maximum size of the ten granular base materials was 19 mm and the amount of fines added on the five materials ranged from 10.7 to 13.8 %. Figures (5) and (6) show the gradations of A-D and W-F materials respectively.
**Figure 5** Grain size distributions for the five A-D granular materials (David et. al. 2009)

**Figure 6** Grain size distributions for the five W-D granular materials (David et. al. 2009)
Table 3(a) and 3(b) show the full name and their physical characteristics respectively for the ten Missouri granular base materials.

**Table 3(a) The list of Ten Missouri Unbound Granular Base Materials (David et. al. 2009)**

<table>
<thead>
<tr>
<th>Granular Base Materials</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jefferson City Dolomite AS-Delivered</td>
<td>JCD A-D</td>
</tr>
<tr>
<td>Jefferson City Dolomite With added Fines</td>
<td>JCD W-F</td>
</tr>
<tr>
<td>Gasconade Dolomite AS-Delivered</td>
<td>Gasc A-D</td>
</tr>
<tr>
<td>Gasconade Dolomite With added Fines</td>
<td>Gasc W-F</td>
</tr>
<tr>
<td>Plattin Limestone AS-Delivered</td>
<td>Plat A-D</td>
</tr>
<tr>
<td>Plattin Limestone With added Fines</td>
<td>Plat W-F</td>
</tr>
<tr>
<td>Winterset Limestone AS-Delivered</td>
<td>Win A-D</td>
</tr>
<tr>
<td>Winterset Limestone With added Fines</td>
<td>Win W-F</td>
</tr>
<tr>
<td>Bethany Falls Limestone AS-Delivered</td>
<td>BF A-D</td>
</tr>
<tr>
<td>Bethany Falls Limestone With added Fines</td>
<td>BF W-F</td>
</tr>
</tbody>
</table>

**Table 3(b) The characteristics of 10 Unbound Granular Materials (David et. al. 2009)**

<table>
<thead>
<tr>
<th>Materials</th>
<th>Water Content [%]</th>
<th>Degree of Saturation [%]</th>
<th>Specific Gravity $G_s$ [g/cm$^3$]</th>
<th>Dry Density $\rho_{dry}$ [g/cm$^3$]</th>
<th>Unit Volume of Aggregate</th>
<th>Unit. Volume of Void</th>
</tr>
</thead>
<tbody>
<tr>
<td>JCD A-D</td>
<td>9.0</td>
<td>80.9</td>
<td>2.472</td>
<td>2.138</td>
<td>0.865</td>
<td>0.135</td>
</tr>
<tr>
<td>JCD W-F</td>
<td>8.9</td>
<td>88.6</td>
<td>2.478</td>
<td>2.179</td>
<td>0.879</td>
<td>0.121</td>
</tr>
<tr>
<td>Gasc A-D</td>
<td>9.4</td>
<td>87.4</td>
<td>2.388</td>
<td>2.139</td>
<td>0.896</td>
<td>0.104</td>
</tr>
<tr>
<td>Gasc W-F</td>
<td>9.1</td>
<td>87</td>
<td>2.408</td>
<td>2.15</td>
<td>0.893</td>
<td>0.107</td>
</tr>
<tr>
<td>Plat A-D</td>
<td>8.0</td>
<td>83.7</td>
<td>2.46</td>
<td>2.142</td>
<td>0.871</td>
<td>0.129</td>
</tr>
<tr>
<td>Plat W-F</td>
<td>7.9</td>
<td>79.6</td>
<td>2.433</td>
<td>2.15</td>
<td>0.884</td>
<td>0.116</td>
</tr>
<tr>
<td>Win A-D</td>
<td>8.6</td>
<td>79.4</td>
<td>2.43</td>
<td>2.138</td>
<td>0.88</td>
<td>0.12</td>
</tr>
<tr>
<td>Win W-F</td>
<td>8.0</td>
<td>80.6</td>
<td>2.404</td>
<td>2.15</td>
<td>0.894</td>
<td>0.106</td>
</tr>
<tr>
<td>BF A-D</td>
<td>7.4</td>
<td>81.3</td>
<td>2.492</td>
<td>2.208</td>
<td>0.886</td>
<td>0.114</td>
</tr>
<tr>
<td>BF W-F</td>
<td>7.0</td>
<td>82.7</td>
<td>2.95</td>
<td>2.229</td>
<td>0.756</td>
<td>0.244</td>
</tr>
</tbody>
</table>
5. Framework Evaluation

Table 4 shows the results of PS porosities ($n_{ps}$) and coordination numbers ($c_{n_{ps}}$) including their resilient modulus ($M_r$) for the nineteen different unbound granular base and subbase materials. The table also includes the volumetric composition of each material. The resilient modulus values were measured by using repeated loading triaxial test (RLTT) with different stress level (Ekblad, 2007; Lekarp, 1999; Austin, 2009 & David et al., 2009). All the resilient modulus values in the table were measured at a mean stress and deviatoric stresses of at 150 kPa.

Table 4 Volumetric composition, PS porosities and Coordination numbers

<table>
<thead>
<tr>
<th>Materials</th>
<th>$V_T$ [%]</th>
<th>$V_v$ [%]</th>
<th>$V_{SS}$ [%]</th>
<th>$V_{agg(&gt;PS)}$ [%]</th>
<th>$n_{ps}$ [%]</th>
<th>$c_{n_{ps}}$</th>
<th>$M_r$ [MPa]</th>
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5.1 Influence of PS Porosity on Resilient Modulus

In Figure 7, it can be observed the relationship between PS porosity and resilient modulus for nineteen unbound materials that contained six materials from Sweden and thirteen materials from USA. In the figure all the Swedish materials (Granite 1,2,3,4 and CONC) except S&G showed PS porosities of less than 50%, implying a continuous stone-to-stone contact structure within the PS. This implies that the volume of SS satisfactorily filled the
voids in the PS (i.e., without disrupting the PS) for these materials. It can also be seen that increasing the porosity of the primary structure resulted a decrease in the resilient modulus.

Accordingly, the resilient modulus for these materials showed relatively high values, in comparison to the Louisiana-derived materials. The exception to this was the Swedish Sand and Gravel (S&G) material, which had no crushed stone fractions. The remaining USA materials (LS, SST, and Gr) showed PS porosities higher than 50%, implying a disrupted PS, resulting generally in lower resilient modulus values than for the Swedish materials having a good PS.

In Figure 7, a similar trend can also be observed for the ten Missouri State materials, i.e. a decreasing resilient modulus with increasing the PS porosity. This clearly shows the effect of PS porosity and gradation for unbound materials composed of the same stone and mineral structure, but with varying amount of fines. For instance, Win A-D and Win W-F exhibited PS porosity of 46.52 % and 54.42 %, having a resilient modulus of 420 MPa and 393 MPa respectively.

Figure 7 Relationship between PS porosity and Resilient Modulus

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5.2 Influence of PS Coordination number (cn_{ps}) on Resilient Modulus

As discussed in Section 3, the coordination number for the primary structure (cn_{ps}) can be determined from Equation (8). The resulting PS coordination numbers (cn_{ps}) for all the materials evaluated are listed in Table 4.

Figure 8 Relationship between average contacts per particle and PS Porosity

Figure 8 shows the PS coordination number as a function of the PS porosity for the nineteen materials evaluated. Granite 1 to 4 and five of the Missouri materials (Plat A-D, Plat W-F, Win A-D, BF-A-D, and BF W-F) showed PS coordination numbers within 6 and 12 which are the coordination numbers of two typical spherical packing arrangements (simple cubic and close cubic/hexagonal).
In Figure 9 one can observe the general trend that an increasing number of PS contact points results in an increase in resilient modulus. It can also be noticed that granular materials having \( c_{n\text{ps}} \) in between 6 and 12 showed relatively high resilient modulus.
6. Conclusions

This paper focused on evaluating the resilient modulus properties of unbound granular materials based on the two key parameters (PS porosity and coordination number) obtained from a new packing-based framework. The primary structure porosity and coordination number parameters can be determined regardless the type of packing arrangement and the system (dimensions) of sieve standards. Nineteen granular materials of different geological and mineralogical origins were used to evaluate the influence of PS porosity and its coordination number on the resilient modulus values of unbound granular materials.

High resilient modulus in the materials was observed for PS porosities in the range between 25.95% and 47.64% and correspondingly also for the associated coordination numbers between 12 and 6 for these PS porosity ranges. This indicates that the resilient modulus property depends on the interaction between the PS particles and their contacts.

In summary, it can be concluded that the new packing-based analysis framework presented is a promising step toward a better understanding of granular material behaviour. It can also be concluded that the relationships presented between the PS porosity and coordination number on one hand versus the resilient modulus on the other show reasonable trends. Finally, the obtained results show that granular materials with PS porosity approximately in between 26% and 50 %, and coordination numbers in the range of 6 to 12 resulted in all cases in resilient modulus values greater than 400 MPa. In comparison, materials outside with PS porosity greater than 50 percent and a coordination number less than 6 always resulted in lower resilient modulus values.
References


Ching S. Chang, Anll Misra, and Sivanuja S. Sundarum, 1991 “Properties of granular packings under low amplitude cyclic loading” Department of Civil Engineering, University of Massachusetts, Amherst, MA 01003, USA.


Isola R. and Dawson A.R. “Aggregate Packing and Coordination Number” Nottingham Centre for Pavement Engineering, University of Nottingham Available: 


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